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(21223) Roll No.

B.Sc.(Com. Sci.)-I Sem.

NP-3573

B.Sc. (Com. Sc.) Examination, Dec.-2023

Applied Mathematics-I

(BCS-102)

Time : Three Hours] *[Maximum Marks : 75*

Note : Attempt **all** the sections as per instructions.

Section-A

(Very Short Answer Type Questions)

Note : Attempt all the **five** questions. Each question carries **3** marks. Very short answer is required not exceeding 75 words. 3×5=15

1. Show that the matrix $A = \frac{1}{\sqrt{2}}\begin{vmatrix} 1 & i \\ -i & -1 \end{vmatrix}$ is unitary.

P.T.O.

2. Find the n^{th} differential coefficient of $e^x \log x$.

3. If $u = \frac{y^2}{2x}$, $v = \frac{x^2+y^2}{2x}$ find $\frac{\partial(u,v)}{\partial(x,y)}$.

4. Show that

$$\int_{x=0}^{1}\int_{y=0}^{2}\int_{z=1}^{2} x^2yz\,dz\,dy\,dx = 1$$

5. Evaluate $\int_c \vec{F}.d\vec{r}$ where $\vec{F}$ is $x^2y^2\hat{i} + y\hat{j}$ and c is $y^2=4x$ in the x-y plane from (0,0) to (4,4).

Section-B

(Short Answer Type Questions)

Note : This section contain three questions. Attempt any **two** questions. Each question carries 7½ marks. 7½×2=15

6. Using matrix theory, prove that the following equations are consistent and solve them :

$x+2y+z=-1$, $6x+y+z=-4$, $2x-3y-z=0$, $-x-7y-2z=7$, $x-y=1$.

7. If $x^x y^y z^z = c$, show that at $x=y=z$,

$$\frac{\partial^2 z}{\partial x \partial y} = -\{x \log(ex)\}^{-1}.$$

8. Change the order of integration in

$$\int_0^a \int_{\sqrt{a^2-y^2}}^{y+a} f(x,y)\,dx\,dy$$

Section-C

(Detailed Answer Questions)

Note : This section contain **five** questions. Attempt **any three** questions. Each question carries 15 marks. 15×3=45

9. Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$$

Hence or otherwise evaluate A^{-1}.

10. (I) Explain $2x^3+7x^2+x-1$ in powers.

(ii) If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that

$$x\left(\frac{\partial u}{\partial x}\right)+y\left(\frac{\partial u}{\partial y}\right)=\frac{1}{2}\tan u$$

11. In any triangle ABC, find the maximum value of cos A cos B cos C by Lagrange's method.

12. (i) Evaluate $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\,dx$

(ii) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

13. (i) If $u=x^2-y^2+4z$, show that $\nabla^2 u = 0$.

(ii) Verify Green's theorem in a plane for

$$\int_C [(3x^2-8y^2)\,dx+(4y-6xy)\,dy]$$

where C is the boundary of the region defined by $z=0$, $y=0$ and $x+y=1$.