

NP-3573(CV-III)
B.Sc. (Computer Science)
Examination, Dec.-2021
APPLIED MATHEMATICS - I
(BCS-102)

Time : 1½ Hours] [Maximum Marks : 75

Note : Attempt questions from all Sections
as per instructions.

Section-A

(Very Short Answer Questions)

Note : Attempt any **two** questions. Each
question carries 7.5 marks.

$$2 \times 7.5 = 15$$

1. Find the differential coefficient of
 $e^{2x} \sin^3 x$.

2. Find the Jacobian $J(u, v)$ for

$$u = e^x \sin y, v = x \log \sin y.$$

3. Find the inverse of the matrix by
elementary transformation.

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

4. Find $\iint_R f(x, y) dA$ for $f(x, y) = 1 - 6x^2y$

and R.I. $0 \leq x \leq 2, -1 \leq y \leq 1$.

5. Find the magnitude of the gradient of the
function $f = xyz^3$ at $(1, 0, 2)$.

Section-B

(Short Answer Questions)

Note : Attempt any **one** question. Each
question carries 15 marks. $1 \times 15 = 15$

6. $u = \sin^{-1} \left(\frac{x^2 + y^2}{x + y} \right)$ show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$$

7. Prove that $x^5 - 5x^4 + 5x^3 - 10$ has a
maximum for $x = 1$, a minimum for $x = 3$
and for $x = 0$, it has neither a maximum

nor a minimum.

8. Reduce the matrix A to canonical form

and find its rank:

$$A = \begin{bmatrix} 2 & -1 & 3 & 3 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 4 \\ 2 & 5 & 11 & 5 \end{bmatrix}$$

Section-C

(Detailed Answer Questions)

Note : Attempt any **two** questions. Each question carries 22.5 marks.

$$2 \times 22.5 = 45$$

9. (a) Find Taylor's Series expansion of $f(xy) = x^3 + xy^2$ about point (2, 1).
(b) Expand $\log(1+x)$ in powers of x.
10. Use the method of Lagrange's multipliers to find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

11. State Cayley-Hamilton theorem. Find the eigen values and eigen vectors of the

matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

12. (a) By changing the order of integration

$$\text{Evaluate } \int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$$

- (b) Define Beta and Gamma function.

Establish the relation between Beta and Gamma function

$$\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

13. (a) Define curl of a vector. Prove the following vector identity:

$$\text{div}(\vec{u} \times \vec{v}) = \vec{v} \cdot \text{curl} \vec{u} - \vec{u} \cdot \text{curl} \vec{v}$$

- (b) State Gauss divergence theorem. Use Gauss divergence theorem in Cartesian form to evaluate

$$\iint_S x \, dy \, dz + y \, dz \, dx + z \, dx \, dy,$$

where the surface S is the sphere

$$x^2 + y^2 + z^2 = a^2.$$