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Roll No:

B. Sc. (Com. Sc.)-I Sem.

NP-3573

B. Sc. (Computer Science) Examination, Dec. 2018

Applied Mathematics-I

(BCS-102)

Time : Three Hours]

[Maximum Marks : 75

Note : Attempt questions from all Sections as per instructions.

Section-A

(Very Short Answer Questions)

Attempt all the five questions. Each question carries 3 marks. Very short answer is required. 3×5=15

1. Find the eigenvalues and eigenvectors of the matrix: 3

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

2. If $z = x^2 \tan^{-1} \frac{y}{x} - y^2 \tan^{-1} \left(\frac{x}{y} \right)$, prove that: 3

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{x^2 - y^2}{x^2 + y^2}$$

(2)

3. If $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \phi$, $z = r \cos \theta$, show that: 3

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin \theta$$

4. Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. 3

5. Find the mass of an octant of the ellipsoid :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

the density at any point being $P = kxyz$. 3

Section-B

(Short Answer Questions)

Attempt any two questions out of the following three questions. Each question carries 7½ marks. Short answer is required. 7½×2=15

6. (a) If $y^{1/m} + y^{-1/m} = 2x$, prove that: 5
 $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$.
 (b) Find the n th derivative of $x^3 e^{ax}$. 2½

7. (a) For what values of λ and μ the following system of equation:

$$2x + 3y + 5z = 9, 7x + 3y - 2z = 8, 2x + 3y + \lambda z = \mu$$

will have (i) unique solution (ii) no solution. 5

- (b) If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1} . 2½

(3)

8. (a) Apply Green's theorem to evaluate

$$\int_C [(2x^2 - y^2)dx + (x^2 + y^2)dy],$$
 where C is the boundary of the area enclosed by the x -axis and the upper half of circle $x^2 + y^2 = a^2$. 5
- (b) Find the work done by a force $y_i + x_j$ which displaces from origin to a point $(i+j)$. 2½

Section-C

(Detailed Answer Questions)

Attempt any *three* questions out of the following five questions. Each question carries 15 marks. Answer is required in detail. 15×3=45

9. (a) Find the rank of the following matrix by reducing it to normal form: 7½

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$

- (b) Find the characteristics equation of the symmetric matrix :

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -2 & 2 \end{bmatrix},$$

and hence find A^{-1} by Cayley-Hamilton theorem. 7½

10. (a) If $y = \tan^{-1} x$ prove that :
 $(1+x^2)y_{n+1} + (2n+1)xy_{n+1} + n(n+1)y_n = 0$
 hence, determine the values of all derivatives of y w. r. t. x when $x=0$. 10

(4)

- (b) If $u = e^{xyz}$, find the value of $\frac{\partial^3 u}{\partial x \partial y \partial z}$. 5
11. (a) Evaluate $\iint_S (yz\hat{i} + zx\hat{j} + xy\hat{k}) \cdot d\vec{s}$, where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant. 7½
- (b) If $\vec{F} = 2zi - xj + yk$, evaluate $\iiint_V \vec{F} \cdot dV$, where V is the region bounded by the surfaces $x=0, y=0, x=2, y=4, z=x^2, z=2$. 7½
12. (a) If $P^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta$, prove that: 10

$$P + \frac{d^2 P}{d\theta^2} = \frac{a^2 b^2}{P^3}.$$
- (b) If $x = a(\cos \theta + \theta \sin \theta)$, $y = a(\sin \theta - \theta \cos \theta)$, find $\frac{d^2 y}{dx^2}$. 5
13. (a) Evaluate $\oint_C \vec{F} \cdot d\vec{r}$ Stokes' theorem, where:

$$\vec{F} = y^2 \hat{i} + x^2 \hat{j} - (x+z) \hat{k}$$
 and C is the boundary of triangle with vertices at $(0, 0, 0), (1, 0, 0)$ and $(1, 1, 0)$. 7½
- (b) Find $\iint_S \vec{F} \cdot \hat{n} dS$, where:

$$\vec{F} = (2x+3z)\hat{i} - (xz+y)\hat{j} + (y^2+2z)\hat{k}$$
 and S is the surface of the sphere having centre $(3, -1, 2)$ and radius 3. 7½