

A  
(21119)

Roll No. ....

Total Questions : 13 ]

[ Printed Pages : 4

**NP-3573**

B.Sc. (Computer Science) Ist Semester  
Examination, Nov., 2019

**APPLIED MATHEMATICS-I**

(BCS-102)

Time : 3 Hrs. ]

[ M.M. : 75

Note :- Attempt questions from all Sections as per instructions.

**Section-A**

3×5=15

Note :- Attempt all five questions.

1. Find the rank of the matrix :

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 4 & 5 & 6 \\ 2 & 3 & 4 \end{bmatrix}$$

2. Verify Euler's theorem for :

$$u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$$

3. Find Jacobian  $J\left(\frac{u, v}{x, y}\right)$  for  $u = x^2, v = y^2$ .

4. Find the area between the parabolas  $y^2 = 4ax$  and  $x^2 = 4ay$ .

5. Show that the vector  $\vec{V} = (x+3y)\hat{i} + (y-3z)\hat{j} + (x-2z)\hat{k}$  is solenoidal.

**Section-B**

7½×2=15

Note :- Attempt any two questions.

6. Find the eigen values and corresponding eigen vectors of the matrix :

$$A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$$

7. If  $y = a \cos(\log x) + b \sin(\log x)$ , then show that :

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$

8. A fluid motion is given by

$$\vec{V} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}.$$

Show that the motion is irrotational and hence find the velocity potential.

ND-100

( 1 )

Turn Over

ND-100

( 2 )

Note :- Attempt any three questions.

9. Verify Cayley-Hamilton theorem for the matrix :

$$A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$$

and hence find  $A^{-1}$ .

10. (a) Expand  $\sin(xy)$  in powers of  $(x - 1)$  and

$\left(y - \frac{\pi}{2}\right)$  as far as terms of second degree.

(b) Change the independent variable  $x$  to  $z$  in the equation :

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = \sin(\log x^2)$$

11. (a) Show that the minimum value of  $u = xy$

$$+ a^3 \left( \frac{1}{x} + \frac{1}{y} \right) \text{ is } 3a^2.$$

(b) Find the point upon the plane  $ax + by + cz = p$  at which the function  $f = x^2 + y^2 + z^2$  has a minimum value and find this minimum  $f$ .

12. Prove that  $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$  also, show that :

$$\int_0^{\pi/2} \sin^p \theta \cos^q \theta d\theta = \frac{\left(\frac{p+2}{2}\right) \times \left(\frac{q+1}{2}\right)}{2 \left(\frac{p+q+2}{2}\right)}$$

13. (a) Using Green's theorem, evaluate

$\int_C (x^2 y dx + x^2 dy)$ , where  $C$  is the boundary described counter clockwise of the triangle with vertices  $(0, 0)$ ,  $(1, 0)$ ,  $(1, 1)$ .

(b) Evaluate  $\oint_C \vec{F} \cdot d\vec{r}$  by Stoke's theorem, where

$\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ , and  $C$  is the boundary of the rectangle  $x = \pm a$ ,  $y = 0$  and  $y = b$ .