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(21223) Roll No. ....

B.Sc. (Com.Sc.)-IIIrd Sem.

NP-3602

B.Sc. (Computer Science) Examination, Dec.-2023

Discrete Structures

(BCS-301)

Time : Three Hours] [Maximum Marks : 75

Note : Attempt **all** the sections as per instructions.

Section-A

Note : Answer **all** questions. Each question carries 3 marks. $3 \times 5 = 15$

1. Prove that union of two countable sets is a countable set.
2. Define Monoid. Also give an example.
3. Define CHAIN. Let X = {1, 2, 3, 4, 6, 12} and the relation $\leq$ be such that $x \leq y$ if x divides y. Draw the Hasse diagram of $(X, \leq)$ and also find a CHAIN in it and locate the CHAIN in Hasse diagram.

P.T.O.

4. Construct the truth table for (p [illegible] q) $\wedge (p \vee r)$.
5. Find the generating function of the following numeric function : $a_r = 2^r - r,\ r \geq 0$

Section-B

Note : Attempt any **two** questions out of the following three questions. Each question carries 7.5 marks.. $2 \times 7.5 = 15$

6. Let Q be the set of rational numbers. Let $f : Q \rightarrow Q$ be defined by $f(x) = 2x+3$, $(x \in Q)$. Show that f is one-one and onto. Also find a formula that defines the inverse function f^{-1}.
7. Given that f=(1 3 2 5) (1 4 3) (2 5 1) is a permutation on fine symbols. Express it as a product of disjoint cycles. Also find the inverse of f and express it as product of disjoint cycles.
8. Prove that two bounded lattices A and B are complemented if and only if $A \times B$ is complemented.

Section-C

Note : Attempt any **three** questions, out of the following five questions. Each question carries 15 marks. $3 \times 15 = 45$

9. Consider the set $N \times N$ the set of ordered pairs of natural numbers. Let R be the relation in $N \times N$ which is defined by (a,b) R (c,d) if and only if $ad = bc$. Prove that R is an equivalence relation. Also show that this relation cannot be a partial order relation.

10. Let R_* be the multiplicative group of all positive real numbers and R be the additive group of all real numbers. Show that the mapping $g : R_* \rightarrow R$ defined by $g(x) = \log x \ \forall x \in R_*$ is an isomorphism.

11. (a) Prove that a subgroup H of a group G is a normal subgroup of G if and only if each left coset of H in G is a right coset of H in G.

(b) If G is a group and H is a subgroup of index 2 in G then prove that H is a normal subgroup of G.

12. (a) Prove that a graph is bipartite if and only if it contains no circuit of odd length.

(b) Prove that an n-vertex simple graph is not bipartite if it has more than $\frac{n^2}{4}$ edges.

13. (a) *Find the minimum number of* elements to be chosen from the set $S = \{1, 2, 3, \ldots, 9\}$ such that two of them should add up to 10.

(b) If Δ is the maximum degree of the vertices in a graph G, then show that chromatic number of $G \leq 1 + \Delta$.